ALAMOUTI CODE

Space Time Block Code

Does NOT need CSI or knowledge of channel coefficients at Transmitter

→ very useful from a Practical viewpoint.

Consider 1×2 MISO System

$r = 1 \quad t = 2$

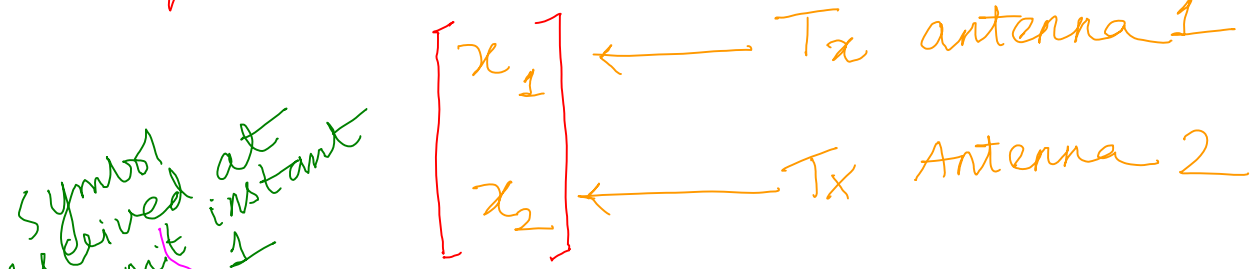
Number of Rx antennas $= 1$

Number of Tx antennas $= 2$

$$y = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w$$

Consider 2 symbols x_1, x_2

First Time instant, Transmit as follows.



Symbol received at instant 1 Transmit

$$y_1 = [h_1 \quad h_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_1$$

Second Transmit Period.

$$\begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix}$$

$-x_2^*$ ← Transmit Ant 1

x_1^* ← Transmit Ant 2

Received symbol after 2nd Transmit Period.

$$y_2 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + w_2$$

$$y_2 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + w_2$$

$$= -h_1 x_2^* + h_2 x_1^* + w_2$$

$$y_2^* = -h_1^* x_2 + h_2^* x_1 + w_2^*$$

$$= h_2^* x_1 + (-h_1^*)(x_2) + w_2^*$$

$$= \begin{bmatrix} h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_2^*$$

$$y_1 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_1$$

$$y_2^* = \begin{bmatrix} h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_2^*$$

$$\underbrace{\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}}_{\bar{y}} = \underbrace{\begin{bmatrix} \overbrace{h_1}^{\bar{G}} & \overbrace{h_2}^{\bar{C}_2} \\ h_2^* & -h_1^* \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\bar{x}} + \underbrace{\begin{bmatrix} w_1 \\ w_2^* \end{bmatrix}}_{\bar{w}}$$

2×1

$$\bar{y} = \bar{c}_1 x_1 + \bar{c}_2 x_2 + \bar{w}$$

$$\bar{c}_1 = \begin{bmatrix} h_1 \\ h_2^* \end{bmatrix} \qquad \bar{c}_2 = \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$$

Gaussian Noise vector

I.I.D. — Independent Identically Distributed.

mean $= 0$

var $= \sigma^2$

$$\bar{c}_1 = \begin{bmatrix} h_1 \\ h_2^* \end{bmatrix} \qquad \bar{c}_2 = \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$$

Observe, that $\bar{c}_1^H \bar{c}_2$

$$= \begin{bmatrix} h_1^* & h_2 \end{bmatrix} \begin{bmatrix} h_2 \\ -h_1^* \end{bmatrix}$$

$$= h_1^* h_2 - h_2 h_1^* = 0$$

$\bar{c}_1, \bar{c}_2$ are orthogonal.

Since $\bar{G}_1$, $\bar{G}_2$ are orthogonal, Alamouti code is also known as,

Orthogonal Space-Time Block Code (OSTBC).

$$\bar{y} = \bar{G}x_1 + \bar{G}_2 x_2 + \bar{W}$$

$$\frac{\bar{G}_1^H}{\|\bar{G}_1\|} \cdot \bar{y} = \frac{\bar{G}_1^H}{\|\bar{G}_1\|}\left(\bar{G}_1 x_1 + \bar{G}_2 x_2 + \bar{W}\right)$$

$$= \frac{\bar{G}_1^H \cdot \bar{G}_1}{\|\bar{G}_1\|} x_1 + \underbrace{\frac{\bar{G}_1^H \bar{G}_2}{\|\bar{G}_1\|}}_{0} x_2 + \frac{\bar{G}_1^H \bar{W}}{\|\bar{G}_1\|}$$

$$= \|\bar{G}_1\| x_1 + 0 + \frac{\bar{G}_1^H \bar{W}}{\|\bar{G}_1\|}$$

$$= \|\bar{G}_1\| x_1 + \underbrace{\frac{\bar{G}_1^H \bar{W}}{\|\bar{G}_1\|}}_{\tilde{W}}$$

$$\tilde{W} = \frac{\bar{G}^H}{\|\bar{G}\|} \cdot \bar{W}$$

$$E\{\bar{W}\bar{W}^H\} = \sigma^2 I$$

$$E\{\tilde{W}\tilde{W}^*\} = E\left\{\frac{\bar{G}^H}{\|\bar{G}\|} \cdot \bar{W}\bar{W}^H \cdot \frac{\bar{G}}{\|\bar{G}\|}\right\}$$

$$= \frac{\bar{G}^H}{\|\bar{G}\|} \cdot E\{\bar{W}\bar{W}^H\} \cdot \frac{\bar{G}}{\|\bar{G}\|}$$

$$= \frac{1}{\|\bar{G}\|^2} \cdot \bar{G}^H \cdot \sigma^2 I \cdot \bar{G} = \sigma^2 \cdot \frac{\|\bar{G}\|^2}{\|\bar{G}\|^2}$$

$$= \sigma^2$$

$$E\{|\tilde{w}|^2\} = E\{\tilde{w}\tilde{w}^*\} = \sigma^2$$

$$\frac{\bar{G}^H}{\|\bar{G}\|} \cdot \bar{y} = \|\bar{G}\| x_1 + \tilde{w}$$

$\tilde{w}$: Gaussian, var $= \sigma^2$

$$SNR = \frac{\|\bar{G}\|^2 \cdot E\{|x_1|^2\}}{\sigma^2}$$

$$\bar{G} = \begin{bmatrix} h_1 \\ h_2^* \end{bmatrix}$$

$$\|\bar{G}\| = \sqrt{|h_1|^2 + |h_2^*|^2}$$

$$= \sqrt{|h_1|^2 + |h_2|^2}$$

$$= \|h\|.$$

$$SNR = \frac{\|\bar{G}\|^2 E\{|x_1|^2\}}{\sigma^2} = \frac{\|h\|^2 E\{|x_1|^2\}}{\sigma^2}$$

Transmit vector = $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

2 symbols in each Transmit vector.

If Tx Power = P.

$$E\{|x_1|^2\} = E\{|x_2|^2\} = \frac{P}{2}$$

Half the Transmit Power.

$$SNR = \frac{\|h\|^2 E\{|x_1|^2\}}{\sigma^2}$$

$$= \frac{\|h\|^2 P/2}{\sigma^2}$$

$$= \frac{1}{2} \cdot \frac{\|h\|^2 P}{\sigma^2}$$

same SNR as MRC except for factor = 1/2